

Background Material Mathematical

Exponents and Logarithms
Graphing Data
Using Spreadsheets

Objectives

- Understand exponents and logarithms; algebraically and graphically
- Understand and use linear (Cartesian) and semi-log graph paper
- Understand and use spreadsheets

Exponents

- Definition:
 $N = b^e$ or $N = b^e = b^{**}e$
- N is the number, b is the base, and e is the exponent
- Examples:
a) $100 = 10^2$ or $100 = 10^2 = 10^{**}2$
b) $100 = e^{4.605}$ or $100 = e^{4.605}$
where **e** is 2.7183 !!!

Exponents: Algebraic

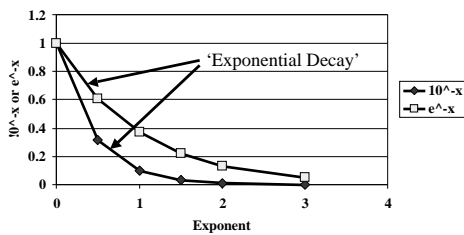
- $10 \times 100 = 10^1 \times 10^2 = 10^{(1+2)} = 10^3 = 1000$
- $10 \times 100 = e^{2.303} \times e^{4.605} = e^{6.908} = 1000$
- Add exponents to multiple numbers
- Subtract exponents for division
- $5.6/1.2 = e^{1.723}/e^{0.182} = e^{(1.723-0.182)} = e^{1.541} = 4.67$
- $23.4/0.264 = e^{??}/e^{??} = e^{(?? - ??)} = e^{??} = ???$

AL

Table of Exponent Values

x	10 ^{-x}	e ^{-x}
0.0	1.000	1.000
0.5	0.316	0.607
1.0	0.100	0.368
1.5	0.032	0.223
2.0	0.010	0.135
3.0	0.001	0.050

Exponents: Graphical



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Logarithms

- Definition
If $N = b^e$ then $e = \log_b N$
- $N =$ number, b is the base, and e is the exponent
- when the base is **10** the logarithm (\log) term is Common log
- when the base is e the logarithm (\ln) is Natural log

Logarithms

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Common logarithms are used for equilibrium calculations, buffer and pH calculations

Natural logarithms are used for pharmacokinetic and other kinetic processes

Logarithms: Algebraic

- $100 = 10^2$ thus $\ln 100 = 2 \ln 10$
and 100 is the same as 10^2
(base 10 assumed)
- Convert from one base to another:
 $\log_a N = \log_b b \cdot \log_b N$ (e.g. $a = e$ and $b = 10$)
 $\ln 100 = \ln 10 \cdot \log 100 = 2.303 \times 2 = 4.606$
- $\ln 23 = \ln 10 \cdot \log 23 = 2.303 \times ???$
 $= ???$

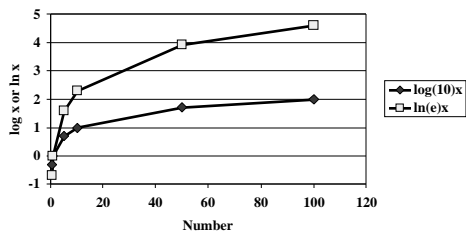
$\ln 23 = \ln 10 \cdot \log 23 = 2.303 \times 1.362 = 3.137$

AL

Table of Logarithm Values

x	$\log_{10}x$	$\ln x$
0.5	-0.301	-0.693
1	0.0	0.0
5	0.699	1.609
10	1.0	2.303
50	1.699	3.913
100	2.0	4.605

Logarithms: Graphical



Graphing Data

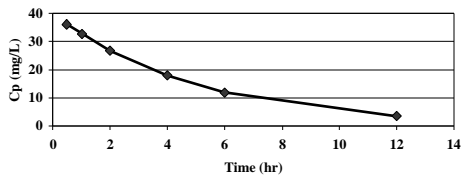
- Graphical representation of data
- Data visualisation
- Data analysis - Parameter estimation

Example Data

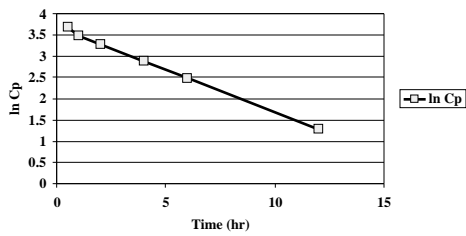
Time (hr)	Cp (mg/L)	ln Cp
0.5	36.2	3.69
1	32.7	3.49
2	26.8	3.29
4	18.0	2.89
6	12.0	2.49
12	3.6	1.29

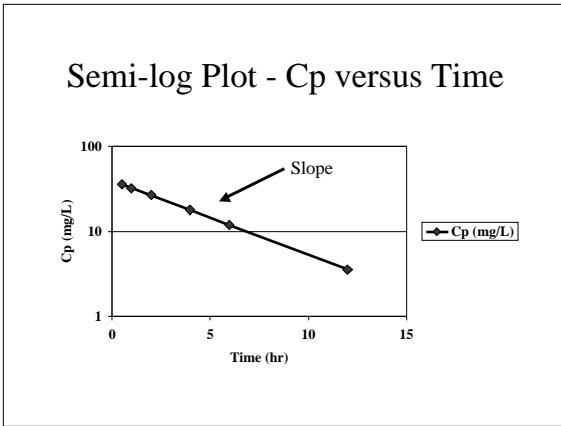
Linear Plot - Cp versus Time

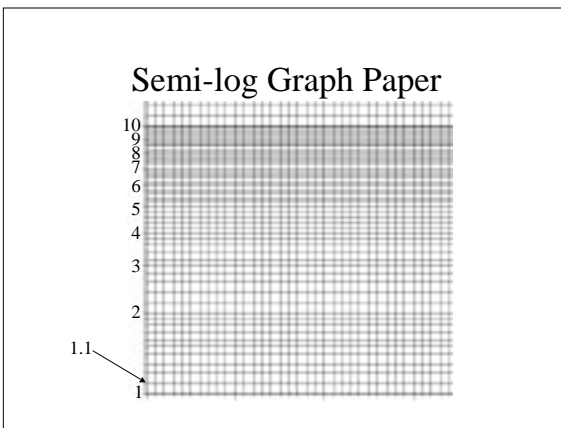
Linear Plot



Linear Plot - ln Cp versus Time

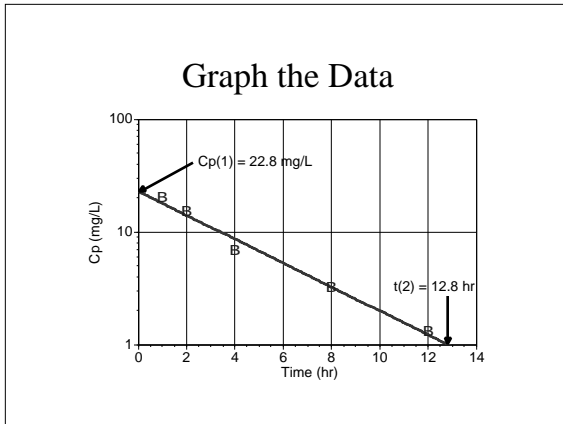






Another Example

Time (hr)	Cp (mg/L)
1	20
2	15
4	6.8
8	3.2
12	1.3



Analyse the Data

- First Point = 0, 22.8
- Second Point = 12.8, 1
- Slope:

$$\frac{\ln y_2 - \ln y_1}{x_2 - x_1} = \frac{\ln 1 - \ln 22.8}{12.8 - 0} = \frac{0.000 - 3.127}{12.8} = -0.244 \text{ hr}^{-1}$$
- $k_{el} = -\text{slope} = 0.244 \text{ hr}^{-1}$
- Parameters: Intercept and Slope

Spreadsheets

- Simulation Calculations
- Rapid 'what-if' Analysis
- Graphing Data
- Parameter Estimation

Simulation - 'What-if' Example

PARAMETERS		
k _{e1}	0.2	hr ⁻¹
C _p (0)	40	mg/L
RESULTS		
Time hr	C _p mg/l	ln C _p
0.0	40.0	3.69
0.5	36.2	3.59
1.0	32.7	3.49
2.0	26.8	3.29
4.0	18.0	2.89
6.0	12.0	2.49
12.0	3.6	1.29

Graphing - Estimation Example

DATA		
A	B	
1	Time (hr)	C _p (mg/L)
2	1	20
3	2	15
4	4	6.8
5	8	3.2
6	12	1.3
7		
8	Slope	-0.2442 hr ⁻¹
9	C _p (0)	22.83 mg/L
10		
11	Graph Data	
12		

Online Resources

- [Graph Paper](#)
- [PHAR 4634 Pages](#)
