

## Intravenous Infusion

---

---

---

---

---

## Objectives

- To understand the diagrams, schemes, and graphs associated with single and multiple infusion regimens
- To recognize the differential and integrated equations associated with single or multiple infusion regimens
- To use the integrated equations to:
  - Determine  $k_{el}$  and  $V$  parameter values
  - Develop safe dosage regimens

---

---

---

---

---

## Intravenous Infusion

- Hospital Setting
- Maintenance therapy
- Stable and compatible drug
- Drugs too toxic for bolus administration
  - E.g. Phenytoin < 50 mg/min
    - Solvent the problem ?

---

---

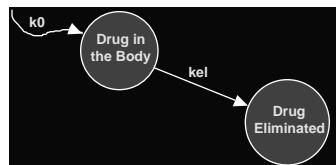
---

---

---

Clinical Pharmacology: An Electronic Drug Reference

### Scheme




---

---

---

---

---

### Differential Equation

$$\frac{V \cdot dC_p}{dt} = k_0 - k_{el} \cdot V \cdot C_p$$

Input +ve  
From infusion      Output -ve  
By Elimination

---

---

---

---

---

### Integration

$$\frac{V \cdot dC_p}{dt} = k_0 - k_{el} \cdot V \cdot C_p$$

$$V \cdot s \cdot \bar{C}_p - C_p(0) = \frac{k_0}{s} - k_{el} \cdot V \cdot \bar{C}_p$$

$$V \cdot \bar{C}_p \cdot (s + k_{el}) = \frac{k_0}{s}$$

$$\bar{C}_p = \frac{k_0}{V \cdot s \cdot (s + k_{el})}$$

---

---

---

---

---

## Back-Transform

$$\frac{C_p}{V} = \frac{k_0}{V \cdot k_{el}} \cdot \frac{1}{1 + e^{-k_{el}t}}$$

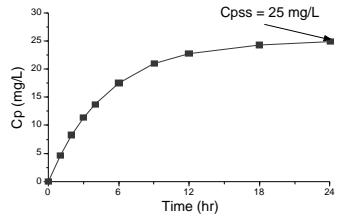
Two roots: 0 and  $-k_{el}$

$$C_p = \frac{k_0}{V \cdot k_{el}} + \frac{k_0}{V \cdot -k_{el}} \cdot e^{-k_{el}t}$$

$$C_p = \frac{k_0}{V \cdot k_{el}} \cdot [1 - e^{-k_{el}t}]$$

Similar to the Equation for Cumulative Amount Excreted Equation

## Continuous Infusion



## Approach to Steady State

$$C_p = \frac{k_0}{k_{el} \cdot V} \cdot [1 - e^{-k_{el}t}]$$

When  $t = \infty$  then  $e^{-k_{el}t} = 0$

$$C_p^{ss} = \frac{k_0}{k_{el} \cdot V}$$

$$\frac{V \cdot dC_p}{dt} = k_0 - k_{el} \cdot V \cdot C_p = 0$$

### Example Calculation

- Data:  $t_{1/2} = 4 \text{ hr}$ ;  $V = 25 \text{ L}$ ;  $C_p^{ss} = 15 \text{ mg/L}$
- Question: What is the required  $k_0$
- $kel = 0.693/4 = 0.17 \text{ hr}^{-1}$
- $k_0 = kel \cdot V \cdot C_p^{ss} = 0.17 \times 25 \times 15 = 63.8 \text{ mg/hr}$
- Using 60 mg/hr
- $C_p^{ss} = 60/(0.17 \times 25) = 14.1 \text{ mg/L}$

---



---



---



---



---



---

### Time to Reach $C_p^{ss}$

- Time to Reach Effective  $C_p - C_p^{ss}$

$$C_p = \frac{C_p^{ss}}{2} = C_p^{ss} \cdot [1 - e^{-kel \cdot t_{half}}]$$

$$\frac{1}{2} = 1 - e^{-kel \cdot t_{half}}$$

$$\frac{1}{2} = e^{-kel \cdot t_{half}} \text{ or } 2 = e^{kel \cdot t_{half}}$$

$$\ln 2 = kel \cdot t_{half} = 0.693 = kel \cdot t_{1/2}$$

$t_{half} = t_{1/2}$  That is, the time it takes to get to half  $C_p^{ss}$  is the elimination half-life

---



---



---



---



---



---

### Time to Reach $C_p^{ss}$

- Approach to  $C_p^{ss}$  is exponential in nature
- However, speed of approach is controlled by  $kel$  NOT  $k_0$
- And, the value of  $C_p^{ss}$  IS controlled or determined by  $k_0$

---



---



---



---



---



---

### Time to Reach $C_p^{ss}$

- 50% to steady state in 1 half-life
- 75% to steady state in 2 half-lives
- 86% to steady state in 3 half-lives
- 94% to steady state in 4 half-lives
- If  $t_{1/2}$  is 4 hours then it will take 16 hours to reach 94% of the desired  $C_p^{ss}$

---



---



---



---



---



---

### Example Calculation

- Data:  $k_0 = 60 \text{ mg/hr}$ ;  $k_{el} = 0.17 \text{ hr}^{-1}$ ;  
 $C_{p_{reqd}} = 10 \text{ mg/L}$ ;  $C_p^{ss} = 14. \text{ mg/L}$
- Question: How long will it take to reach  $C_{p_{reqd}}$

$$C_p = \frac{k_0}{k_{el} \cdot V} \cdot [1 - e^{-k_{el}t}]$$

$$10 = \frac{60}{0.17 \times 25} \times [1 - e^{-0.17xt}]$$

$$0.292 = e^{-0.17xt} \quad \text{OR} \quad t = 7.24 \text{ hr}$$

---



---



---



---

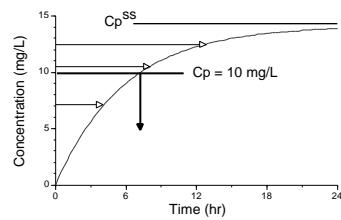


---



---

### Time to Reach $C_p^{ss}$




---



---



---



---



---



---

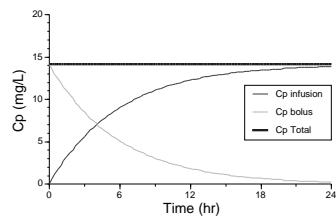
## Strategies for Faster Approach to $C_p^{ss}$

- Combine Bolus and Infusion Administration
  - When a bolus dose is possible
- Fast then Slow Infusion Regimen

## Bolus Dose plus Infusion

- Data:  $C_p^{ss} = 14.1 \text{ mg/L}$ ;  $V = 25 \text{ L}$ ; and  $k_{el} = 0.17 \text{ hr}^{-1}$
- Loading Dose  
 $\text{Dose} = V \cdot C_p^{ss} = 25 \times 14.1 = 353 \text{ mg}$
- Maintenance Dose from before  
 $k_0 = 60 \text{ mg/hr}$

## Bolus Dose plus Infusion



### Fast and Slow Infusion

- Data:  $C_p^{ss} = 14.1 \text{ mg/L}$ ;  $k_0 = 60 \text{ mg/hr}$ ;  $V = 25 \text{ L}$ ; and  $k_{el} = 0.17 \text{ hr}^{-1}$
- Required Fast Infusion over 30 minutes

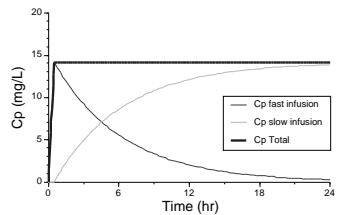
$$C_p = \frac{k_0}{k_{el} \cdot V} \cdot [1 - e^{-k_{el}t}]$$

$$14.1 = \frac{k_0}{0.17 \times 25} \times [1 - e^{-0.17 \times 0.5}]$$

$$k_0 = 735 \text{ mg/hr}$$

- Slow Infusion from before  
 $k_0 = 60 \text{ mg/hr}$

### Fast and Slow Infusion



### Fast and Slow Infusion

- Dosage Regimen  
 $k_0 = 735 \text{ mg/hr}$  for 30 minutes  
 $k_0 = 60 \text{ mg/hr}$
- NOTE:  $C_p^{ss}$  when  $k_0 = 735 \text{ mg/hr}$

$$C_p^{ss} = \frac{k_0}{k_{el} \cdot V} = \frac{735}{0.17 \times 25} = 173 \text{ mg/L}$$

Toxic Overdose

## Once the Infusion Stops

- Schemes
- Equations
- Figures

---



---



---



---

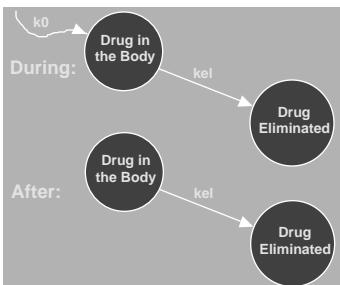


---



---

## Schemes




---



---



---



---



---



---

## Equations

- Differential Equation

$$\frac{dC_p}{dt} = -k_{el} \cdot C_p$$

Concentration at  
the End of the  
Infusion

- Laplace Transform

$$s \cdot \bar{C}_p - \frac{k_0}{k_{el} \cdot V} \cdot [1 - e^{-k_{el} \cdot T}] = -k_{el} \cdot \bar{C}_p$$

$$\bar{C}_p = \frac{k_0}{k_{el} \cdot V} \cdot [1 - e^{-k_{el} \cdot T}] \cdot \frac{1}{(s + k_{el})} = \frac{C_p^T}{(s + k_{el})}$$

---



---



---



---



---



---

### Equation (continued)

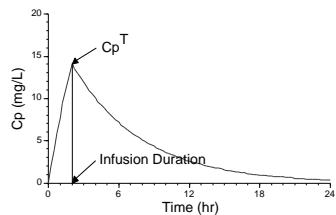
$$\frac{C_p}{C_p^T} = \frac{k_0}{k_{el} \cdot V} e^{-k_{el} \cdot T} + \frac{1 - e^{-k_{el} \cdot T}}{k_{el} \cdot V}$$

Take the Back Transform

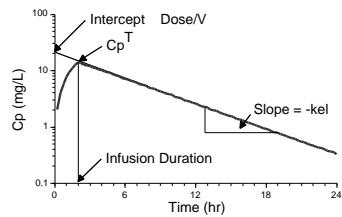
$$C_p = C_p^T \cdot e^{-k_{el} \cdot (t-T)} = \frac{k_0}{k_{el} \cdot V} \cdot [1 - e^{-k_{el} \cdot T}] \cdot e^{-k_{el} \cdot (t-T)}$$

For  $t < T$  then  $T = t$   
 For  $t > T$  then  $T = T$

### Linear Plot



### Semi-log Plot

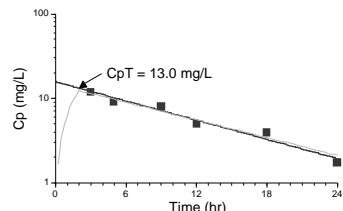


### Example Calculation

Infusion Rate = 100 mg/hr for 2 hours

Time (hr)	Cp (mg/L)
3	12
5	9
9	8
12	5
18	3.9
24	1.7

### Example Calculation



### Elimination Rate Constant

$$k_{el} = \frac{\ln C_{p_1} - \ln C_{p_2}}{t_2 - t_1}$$

$$k_{el} = \frac{\ln 13 - \ln 1.9}{24 - 2} = 0.087 \text{ hr}^{-1}$$

## Apparent Volume of Distribution

$$C_p^T = \frac{k_0}{k_{el} \cdot V} \cdot [1 - e^{-k_{el} \cdot t}]$$

$$13 = \frac{100}{0.087 \times V} \cdot [1 - e^{-0.087 \times 2}]$$

$$V = 88.4 \times [1 - 0.840] = 14.1 \text{ L}$$

---



---



---



---



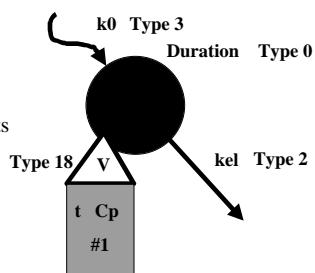
---



---

## Data Analysis

- Using Boomer
  - Define Model
  - Enter Data
  - Review Results




---



---



---



---

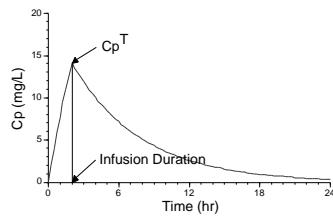


---



---

## Time Interrupts




---



---



---



---



---



---

## Data Analysis

- Using Boomer
  - Define Model
  - Enter Data
  - Review Results

---

---

---

---

---

## Data Analysis

- Using Boomer
  - Define Model
  - Enter Data
  - Review Results

---

---

---

---

---

## Data Analysis

- Using SAAM II
  - Define Model
  - Enter Data
  - Review Results

---

---

---

---

---

## Data Analysis

- Using SAAM II
  - Define Model
  - Enter Data
  - Review Results

---

---

---

---

---

## Data Analysis

- Using SAAM II
  - Define Model
  - Enter Data
  - Review Results

---

---

---

---

---

## HyperCard Stack

- One Compartment IV Infusion

---

---

---

---

---

## Objectives

- To understand the diagrams, schemes, and graphs associated with single and multiple infusion regimens
- To recognize the differential and integrated equations associated with single or multiple infusion regimens
- To use the integrated equations to:
  - Determine  $k_{el}$  and  $V$  parameter values
  - Develop safe dosage regimens

---

---

---

---

---

---